

# Schedulability Analysis for non Necessarily Harmonic Real-Time Systems with Precedence and Strict Periodicity Constraints using the Exact Number of Preemptions and no Idle Time

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## 1 Introduction

Scheduling theory as it applies to hard real-time environments with precedence and strict periodicity constraints — environments where the failure to satisfy any constraint may have disastrous consequences [1], [2] — seems currently to be enjoying a renaissance. The most widely studied problems in this field concern periodic non-preemptive tasks for systems where data are collected through captors and the control is performed through actuators [3], [4]. The tasks corresponding to captors and actuators must satisfy a strict periodicity constraint. For reasons of consistency and predictability, we assume in this paper that all tasks are subject to this constraint. The domains considered here include automobiles, avionics, mobile robotics, telecommunications, etc. Although preemptive scheduling algorithms are able to successfully schedule some systems that cannot be scheduled by any non-preemptive scheduling algorithm, the cost of preemption may not be negligible. Therefore, when preemption is allowed, its exact cost has to be explicitly considered in the schedulability conditions in order to avoid wasting resources and provide safety in terms of guaranteeing the right behavior of the system at run-time. In this paper, we address the scheduling problem of hard real-time systems composed of **dependent, strictly periodic**, preemptive tasks in the monoprocessor case. The strictly periodic constraint implies that, for such a system, any task starts its execution at the beginning of its period whereas the dependence constraint implies that any task cannot start its execution before the end of another task preceding it. We assume here that no jitter is allowed at the beginning of each task. To clearly distinguish between the specification level and its associated model, we shall use the term *operation* rather than the commonly used “*task*” [5] which is too closely related to the implementation level.

For systems with the above-mentioned constraints, in [6] we proved that some of them can be eliminated because they are definitely not schedulable, then we solved the problem

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for systems with harmonic periods<sup>1</sup> in [7]. Here, we first generalize these results to the case of systems with periods that are not necessarily harmonic. Then, we provide a necessary and sufficient schedulability condition which takes into account the exact number of preemptions for a system with such constraints when no idle time is allowed. That means the processor always executes an operation if there is one to execute. Indeed, even though the cost  $\alpha$  of one preemption — the context switching time including the storage as well as the restoration of the context that the processor needs when a preemption occurs — is easy to know for a given processor, it remains a challenging problem to count the exact number of preemptions of each instance for a given operation [8], [9], [7]. As in [10], we consider only predictable processors without cache or complex internal architecture. We consider a set of  $n$  strictly periodic preemptive operations  $\tau_i, 1 \leq i \leq n$  with precedence constraints. Each operation  $\tau_i$  is an infinite sequence of instances<sup>2</sup>  $\tau_i^k, k \in \mathbb{N}^+$ , and is characterized by a Worst Case Execution Time (WCET)  $C_i$ , not including any approximation of the cost of preemption, as is usually the case in the classical real-time scheduling theory [11], [12], [13], [14], and a period  $T_i$ . In each instance  $\tau_i^k$ , operation  $\tau_i$  has to execute for  $C_i$  time units. The context switching time corresponding to the activation as well as the termination of an operation is constant, and thus is included in the WCET. Regarding the constraints, we have the following information.

The **precedence** constraint is given by a partial order on the execution of the operations. An operation  $\tau_i$  preceding an operation  $\tau_j$  is denoted by  $\tau_i \prec \tau_j$  which means that  $s_i^k \leq s_j^k, \forall k \geq 1$  thanks to the result given in [4],  $s_i^k$  denotes the start time of  $\tau_i^k$ . In that paper it was proved that given two operations  $\tau_i = (C_i, T_i)$  and  $\tau_j = (C_j, T_j)$ :  $\tau_i \prec \tau_j \implies T_i \leq T_j$ . Consequently, the operations must be scheduled in an increasing order of their periods corresponding to classical fixed priorities, using Rate Monotonic (RM), where the shorter the period the higher the priority [7], [4]. We re-index operations in such a way that  $\tau_1 \prec \tau_2 \prec \dots \prec \tau_n$ , that is to say  $\tau_1$  precedes  $\tau_2$ ,  $\tau_2$  precedes  $\tau_3$  and so on. In the context of this paper we shall use the term “level” rather than priority, level 1 which corresponds to operation  $\tau_1$  being the highest, and level  $n$  which corresponds to operation  $\tau_n$  being the lowest.

The **strict periodicity** constraint means that the start times  $s_i^k$  and  $s_i^{k+1}$  of two consecutive instances corresponding to operation  $\tau_i$  are **exactly** separated by its period:  $s_i^{k+1} - s_i^k = T_i, \forall k \geq 1$ . The instance started at time  $s_i^1 + kT_i$  has  $s_i^1 + (k+1)T_i$  as its deadline.

For such a system of operations with precedence and strict periodicity constraints, we propose a method to compute on the one hand the exact number of preemptions, and on the other hand the schedule of the system when no idle time is allowed, i.e. the processor will always execute an operation as soon as it is possible to do so. Although idle time may help the system to be schedulable, when no idle time is allowed it is easier to find the start times of all the instances of an operation according to the precedence relation.

For the sake of readability and without any loss of generality, from now on, although it is not entirely realistic, we will consider the cost of one preemption for the processor to be  $\alpha = 1$  time unit. It is worth noticing that the analysis performed here would work even if the preemption cost were not a constant.

The remainder of the paper is structured as follows: section 2 describes the model and gives the notations used throughout this paper. Section 3 provides the definitions we need to take into account the exact number of preemptions in the schedulability analysis presented in section 4. That section explains in detail, on the one hand, our scheduling algorithm

<sup>1</sup> A sequence  $(a_i)_{1 \leq i \leq n}$  is harmonic if and only if there exists  $q_i \in \mathbb{N}$  such that  $a_{i+1} = q_i a_i$ . Notice that we may have  $q_{i+1} \neq q_i \quad \forall i \in \{1, \dots, n\}$ .

<sup>2</sup> Throughout the paper all subscripts refer to operations whereas all superscripts refer to instances.

which counts the exact number of preemptions and, on the other hand, derives the new schedulability condition. We conclude and propose future work in section 5.

## 2 Model

Throughout the paper, we assume that all timing characteristics are non-negative integers, i.e. they are multiples of some elementary time interval (for example the ‘‘CPU tick’’, the smallest indivisible CPU time unit). We denote by  $\tau_i = (C_i, T_i)$ : an operation,  $T_i$ : Period of  $\tau_i$ ,  $C_i$ : WCET of  $\tau_i$  without any preemption approximation,  $C_i \leq T_i$ ,  $\alpha$ : Temporal cost of one preemption for a given processor,  $\tau_i^k$ : The  $k^{\text{th}}$  instance of  $\tau_i$ ,  $N_p(\tau_i^k)$ : Exact number of preemptions of  $\tau_i$  in  $\tau_i^k$ ,  $C_i^k = C_i + N_p(\tau_i^k) \cdot \alpha$ : Preempted Execution Time (PET) of  $\tau_i$  including its exact preemption cost in  $\tau_i^k$ ,  $s_i^1$ : Start time of the first instance of  $\tau_i$ ,  $s_i^k = s_i^1 + (k-1)T_i$ : Start time of  $\tau_i^k$ ,  $R_i^k$ : Response time of  $\tau_i^k$ .

A *valid schedule*  $S$  for the system taking into account the exact number of preemptions will be yielded by the set of start times of the first instance for all operations:  $S = \{(s_1^1, s_2^1, \dots, s_n^1)\}$ . Since all the operations except the one with the shortest period w.r.t. the precedence relations may be preempted, the execution time of an operation may vary from one instance to another due to the number of preemptions. Therefore, the *preempted execution time* (PET) [10] which corresponds to the WCET augmented with the exact cost due to preemptions for each instance of an operation may also vary from one instance to another. Consequently, the PET denoted  $C_i^k$  for instance  $\tau_i^k$  depends on the instance and on the number of preemptions occurring in that instance. Its computation will be detailed below.

Because we intend to take into account the exact number of preemptions, and because all operations may be preempted, except the first one, i.e. the one with the shortest period, all instances of all operations must be considered since the number of preemptions may be different from one instance to another. We give a schedulability condition for each operation individually according to operations with shorter periods. For each operation, our scheduling algorithm first provides the start time of the first instance, then computes the exact number of preemptions per instance. This individual operation analysis leads, at the end, to a schedulability condition for all operations.

It has been shown in [4], [7] that systems with precedence and strict periodicity constraints repeat identically after a time called the *hyperperiod* which corresponds to the Least Common Multiple (LCM) of the periods of all the operations.

## 3 Definitions

All the definitions and terminologies used in this section are directly inspired by [10] and are applied here to the case of a model with precedence and strict periodicity constraints.

From the point of view of any operation  $\tau_i$ , we define the *hyperperiod at level  $i$* ,  $H_i$ , which is given by  $H_i = LCM\{T_j\}_{\tau_j \in sp(\tau_i)}$ , where  $sp(\tau_i)$  is the set of operations with a period shorter than or equal to that of operation  $\tau_i$ . The set  $sp(\tau_i)$  may include  $\tau_i$ . It is obvious that  $H_i$  time units after the first start time  $s_i^1$  of operation  $\tau_i$ , the start time of the next instance is exactly the same as that of  $s_i^1$  w.r.t. the start time of the first instances of operations preceding  $\tau_i$ . This characteristic derives from both the precedence and the strict periodicity constraints. Without any loss of generality we assume that the first operation  $\tau_1$  starts its execution at time  $t = 0$ . Since at each level  $i$  the schedule of  $\tau_i$  repeats indefinitely, it is sufficient to perform the scheduling analysis in the interval  $[s_i^1, s_i^1 + H_i]$  for  $\tau_i$  and  $[0, s_n^1 + H_n]$  for the

whole set of operations. Therefore,  $\tau_i$  starts  $\sigma_i$  times in each hyperperiod at level  $i$  starting from 0, with  $\sigma_i = \frac{H_i}{T_i} = \frac{LCM\{T_j\}_{\tau_j \in sp(\tau_i)}}{T_i}$ .

Because operation  $\tau_i$  may only be preempted by the set of operations with a period shorter than  $\tau_i$  denoted  $sp(\tau_i)$ , then there are exactly  $\sigma_i$  different PETs for operation  $\tau_i$ . In other words, from the point of view of any operation  $\tau_i$ , we can define the function  $\pi$ , inflating  $C_i$ , as  $\pi: \mathbb{N}^+ \times \mathbb{N}^+ \rightarrow \mathbb{N}^{+\sigma_i} \times \mathbb{N}^+$ , where  $\pi(C_i, T_i) = \pi(\tau_i) = ((C_i^1, C_i^2, \dots, C_i^{\sigma_i}), T_i)$ , which maps the WCET  $C_i$  of operation  $\tau_i$  into its respective PET  $C_i^k$  in each instance  $\tau_i^k$  when  $\tau_i$  is schedulable.

Because of the precedence constraints among operations and because we proceed the schedule from the operation with the shortest period towards the operation with the longest period. At each priority level the goal is to fill available time units in the previous schedule thus far obtained, with slices of the WCET of the current operation taking into account the exact number of preemptions, and hence we obtain the next current schedule. Consequently, we represent the previous schedule of every instance  $\tau_i^k$  of the current operation  $\tau_i = (C_i, T_i)$  by an ordered set of  $T_i$  time units where some are already executed because of the execution of operations with shorter periods relatively to  $\prec$ , and the others are still available for the execution of operation  $\tau_i$  in that instance. We call this ordered set which describes the state of each instance  $\tau_i^k$  the  $\mathcal{M}_i^k$   $T_i$ -mesoid. We denote a time unit already executed by an “ $e$ ” and a time unit still available by an “ $a$ ”. The switch from an  $a$  to an  $e$  represents a preemption if the WCET of the current operation is strictly greater than the cardinal of the sub-set corresponding to the first sequence of  $a$ . Depending on the remaining execution time while filling available time units, this situation may occur again leading therefore to several preemptions which themselves may result in causing others. The cardinal of a sub-set corresponding to a sequence of consecutive time units already executed is called a *consumption*. It will be denoted by its value inside brackets. We enumerate the sequence of available time units according to natural numbers. This enumeration is done from the end of the first sequence of time units already executed in that instance. Each of these natural numbers corresponds to the number of available time units since the end of the first consumption. They represent all the possible PETs of the operation under consideration in the corresponding instance. Each of these natural numbers  $a_i$  is called an *availability*. For example, the 13-mesoid  $\{e, e, e, a, a, a, e, e, a, a, e, a, a\}$  will be represented by  $\{(3), 1, 2, 3, (2), 4, 5, (1), 6, 7\}$ ,  $(3), (2), (1)$  are consumptions and  $1, 2, 3, 4, 5, 6, 7$  are availabilities. More details on the definition of a  $T_i$ -mesoid are given in [10].

From the point of view of the current operation  $\tau_i = (C_i, T_i)$ , there are as many  $T_i$ -mesoids as instances in the hyperperiod  $H_i$  at level  $i$ . Therefore, there are  $\sigma_i$   $T_i$ -mesoids in  $H_i$  which will form a sequence of  $T_i$ -mesoids. We call  $\mathcal{L}_i^b = \{\mathcal{M}_i^{b,1}, \mathcal{M}_i^{b,2}, \dots, \mathcal{M}_i^{b,\sigma_i}\}$  the sequence of  $\sigma_i$   $T_i$ -mesoids **before**  $\tau_i$  is scheduled in the current schedule. The process used to build the sequence  $\mathcal{L}_i^b$  of operation  $\tau_i$  will be detailed later. We define for each  $T_i$ -mesoid  $\mathcal{M}_i^{b,k}$  the corresponding *universe*  $X_i^k$  to be the ordered set, compatible with that of the corresponding mesoid, which consists of all the availabilities of  $\mathcal{M}_i^{b,k}$ . That is to say, all the possible values that  $C_i^k$  can take in  $\mathcal{M}_i^{b,k}$ . Recall that  $C_i^k$  denotes the PET of  $\tau_i$  in  $\tau_i^k$ , the  $k^{th}$  instance of  $\tau_i$ . Operation  $\tau_i$  will be said to be *potentially schedulable* if and only if

$$\begin{cases} C_i \in X_i^k & \forall k \in \{1, \dots, \sigma_i\} \\ \mathcal{M}_i^{b,k} \text{ starts with an available time unit} & \text{for each } k \in \{1, \dots, \sigma_i\} \end{cases} \quad (1)$$

The first  $\sigma_i$  equations of (1) verify that  $C_i$  belongs to each universe at level  $i$ . Then, the next  $\sigma_i$  equations verify that every  $\mathcal{M}_i^{b,k}$  starts with an availability as no idle time is allowed.

These verifications are necessary for the strict periodicity constraints to be satisfied. As a matter of fact, if a  $T_i$ -mesoid starts with a consumption it is not possible to fill the previous schedule with slices of the WCET of the current operation  $\tau_i$  taking into account the cost of preemption as it belongs to a lower level than those already scheduled w.r.t  $\prec$ . Therefore its start time is postponed to the end of the consumption in the previous schedule, and thus does not satisfy the strict periodicity constraint of  $\tau_i$ . In this case the system is not schedulable. Notice that when this situation arises the three non schedulability conditions given in [6] hold. Since  $C_i \in \{1, 2, \dots, T_i\}$ ,  $\forall 1 \leq i \leq n$ , let us define the following binary relation on each instance.

$$\mathcal{R}: \text{“availability } a_{i_1} \text{ leads to the same number of preemptions as availability } a_{i_2}\text{”}, \\ a_{i_1}, a_{i_2} \in \{1, 2, \dots, T_i\}$$

$\mathcal{R}$  is clearly an equivalence relation on  $\{1, 2, \dots, T_i\}$  (reflexive, symmetric, transitive). Now, since  $X_i^k \subseteq \{1, 2, \dots, T_i\}$ ,  $\forall 1 \leq k \leq \sigma_i$ , thus  $\mathcal{R}$  is also an equivalence relation on  $X_i^k$  and each  $X_i^k$  together with  $\mathcal{R}$  is a *setoid*<sup>3</sup>. From now on, we consider only the restriction of  $\mathcal{R}$  on  $X_i^k$ ,  $k = 1, \dots, \sigma_i$  because  $X_i^k$  represents all the available time units in instance  $\tau_i^k$ .

Each  $T_i$ -mesoid consists of a sequence of time units already executed, i.e. consumptions, due to the schedule of operations with shorter periods, followed or preceded by a sequence of time units still available, i.e. availabilities. Since each switch from an available time unit to an already executed time unit possibly corresponds to a preemption, then according to the value of  $C_i$  several preemptions may occur. Among the possible values that  $C_i$  can take, those which will lead to the same number of preemptions will be said to be equivalent w.r.t. to  $\mathcal{R}$ , and thus will belong to the same *equivalence class*. Therefore, the *equivalence classes* of each universe correspond to the subsets of availabilities determined by two consecutive consumptions in the associated mesoid. The start time of the first instance  $s_i^1$  of operation  $\tau_i$  occurs at least after the end time of that of operation  $\tau_{i-1}$  in order to satisfy the strict periodicity constraint. Moreover,  $s_i^1$  occurs as soon as possible since no idle time is allowed. The latter statement implies that operation  $\tau_i$  starts  $\Delta_{i-1}$  time units after the start time  $s_{i-1}^1$  of the first instance of operation  $\tau_{i-1}$ . The computation of  $\Delta_{i-1}$  will be detailed later on. Already, it is worth noticing that  $\Delta_{i-1}$  is longer than or equal to the response time of  $\tau_{i-1}$  in its first instance because when the last piece of the PET of  $\tau_{i-1}$  fits exactly a sequence of consecutive availabilities, then the start time of the first instance of  $\tau_i$  is postponed. Hence, we can derive the first start time of any potentially schedulable operation  $\tau_i$  as the **sum** of the start time  $s_{i-1}^1$  of the first instance of operation  $\tau_{i-1}$  and  $\Delta_{i-1}$ :  $s_i^1 = s_{i-1}^1 + \Delta_{i-1}$ .

When equation (1) holds for a given operation  $\tau_i$ , we call  $\mathcal{L}_i^a = \{\mathcal{M}_i^{a,1}, \mathcal{M}_i^{a,2}, \dots, \mathcal{M}_i^{a,\sigma_i}\}$  the sequence of  $\sigma_i$   $T_i$ -mesoids of operation  $\tau_i$  **after**  $\tau_i$  is scheduled.  $\mathcal{L}_i^a$  is a function of  $\mathcal{L}_{i-1}^b$  which itself is a function of  $\mathcal{L}_{i-1}^a$ , both detailed as follows. Thanks to everything we have presented up to now, we can assume without any loss of generality that the start time of the first instance of the operation with the shortest period, here  $\tau_1$ , starts its execution at time  $t = 0$ , i.e.  $s_1^1 = 0$ .

Let  $f$  be the function such that  $\mathcal{L}_i^b = f(\mathcal{L}_{i-1}^a)$  which transforms the sequence  $\mathcal{L}_{i-1}^a$  of  $\sigma_{i-1}$   $T_{i-1}$ -mesoids after operation  $\tau_{i-1}$  has been scheduled at level  $i-1$  into the sequence  $\mathcal{L}_i^b$  of  $\sigma_i$   $T_i$ -mesoids before operation  $\tau_i$  is scheduled at level  $i$ .

As mentioned above, a mesoid consists only of time units already executed denoted by “e” and time units still available denoted by “a”. Moreover, the cardinal of a mesoid is equal to the period of the operation under consideration whatever the level is. As such, the

<sup>3</sup> A setoid is a set equipped with an equivalence relation.

function  $f$  transforms a time unit already executed (resp. still available) in the sequence  $\mathcal{L}_{i-1}^a$  into a time unit already executed (resp. still available) in the sequence  $\mathcal{L}_i^b$  by following an index  $\psi$  which enumerates according to natural numbers, the time units (already executed or still available) in the sequence  $\mathcal{L}_{i-1}^a$  of operation  $\tau_{i-1}$  after  $\tau_{i-1}$  is scheduled.  $\psi$  starts from the first available time unit of the first mesoid  $\mathcal{M}_{i-1}^{a,1}$  towards the last time unit of the last mesoid  $\mathcal{M}_{i-1}^{a,\sigma_{i-1}}$ , and then circles around to the beginning of the first mesoid  $\mathcal{M}_{i-1}^{a,1}$  again, until we get the  $\sigma_i$   $T_i$ -mesoids of  $\mathcal{L}_i^b$ . During this process each time  $\psi = T_i$ , a  $T_i$ -mesoid is obtained for the sequence  $\mathcal{L}_i^b$  and then the next  $T_i$ -mesoid is obtained by starting to count again from the next time unit to the current one. Indeed, the previous schedule at level  $i$  (the schedule obtained at level  $i-1$ ) consists of  $H_{i-1}$  time units whereas the schedule of the current operation  $\tau_i$  is computed upon  $H_i$  time units after the start time of its first instance  $s_i^1$ . That amounts to extending the previous schedule from  $H_{i-1}$  to  $H_i$  time units by identically repeating the previous schedule as often as necessary to obtain  $H_i$  time units.

Due to the precedence and strict periodicity constraints, notice that  $\psi$  in contrast to index  $\zeta$  used in [10] which started from the first time unit, starts from the first available unit of the first  $T_{i-1}$ -mesoid as no idle time is allowed. The value of  $\Delta_{i-1}$  is therefore the consumption before the first available time unit in the sequence  $\mathcal{L}_{i-1}^a$  of operation  $\tau_{i-1}$ .

Since  $\tau_1$  is the operation with the shortest period,  $sp(\tau_1) = \{\tau_1\}$  and thus  $\sigma_1 = \frac{H_1}{T_1} = 1$ .

Moreover, because  $\tau_1$  is never preempted, we have  $\mathcal{L}_1^b = \{\mathcal{M}_1^{b,1}\} = \{\{1, 2, \dots, T_1\}\}$  and  $\mathcal{L}_1^a = \{\mathcal{M}_1^{a,1}\} = \{\{(C_1), 1, 2, \dots, T_1 - C_1\}\}$ .

Let  $g$  be the function such that  $\mathcal{L}_i^a = g(\mathcal{L}_i^b)$  which transforms the sequence  $\mathcal{L}_i^b$  of  $\sigma_i$   $T_i$ -mesoids before operation  $\tau_i$  has been scheduled at level  $i$  into the sequence  $\mathcal{L}_i^a$  of  $\sigma_i$   $T_i$ -mesoids after operation  $\tau_i$  has been scheduled at level  $i$ .

For each  $T_i$ -mesoid  $\mathcal{M}_i^{b,k}$  of  $\mathcal{L}_i^b$  we compute the PET  $C_i^k$  that we add to all the consumptions appearing in that  $T_i$ -mesoid before the availability corresponding to that PET. This yields the response time  $R_i^k$  of operation  $\tau_i$  in instance  $\tau_i^k$ . The PET is computed by using a fixed-point algorithm which is detailed in the next section. Now, for each  $T_i$ -mesoid of  $\mathcal{L}_i^b$ , function  $g$  transforms a time unit already executed in the sequence  $\mathcal{L}_i^b$  into a time unit already executed in the sequence  $\mathcal{L}_i^a$ , and transforms a time unit still available into either a time unit still available or a time unit already executed w.r.t. the following condition. We use an index which enumerates using numerals the time units from the first to the last one in each  $T_i$ -mesoid  $\mathcal{M}_i^{b,k}$  of  $\mathcal{L}_i^b$ . If the current value of the index is less than or equal to  $R_i^k$ , then function  $g$  transforms the time unit still available into a time unit already executed due to the execution of instance  $\tau_i^k$ , otherwise  $g$  transforms it into a time unit still available. Indeed, function  $g$  fills available time units in the current schedule with slices of the PETs in each  $T_i$ -mesoid, leading to the previous schedule for the next operation at level  $i+1$  w.r.t.  $\prec$ .

#### 4 The proposed approach

In this section, we outline our approach for a system with an arbitrary number of operations. This approach leads to a new schedulability condition for hard real-time systems with precedence and strict periodicity constraints using the exact number of preemptions and no idle time allowed. This condition is new in the sense that it takes into account the exact number of preemptions in the schedulability analysis for such systems rather than using an approximation in the WCETs.

Since the schedule proceeds from the operation with the shortest period corresponding to the highest level, to the one with the longest period corresponding to the lowest level, then for every potentially schedulable operation, we determine its schedule thanks to those with shorter periods. At each priority level  $i$ , the basic idea consists in filling availabilities in each mesoid of the sequence  $\mathcal{L}_i^b$ , before operation  $\tau_i$  is scheduled, with slices (cardinal of equivalence classes) of its inflated WCET while taking into account the cost of the exact number of preemptions. At each preemption occurrence,  $\alpha$  time units add to the remaining execution time of the instance of the operation under consideration. This situation may occur again w.r.t. the remaining execution time, leading therefore to several preemptions which themselves may cause others. This is why it is crucial to calculate the exact number of preemptions. Finally, we obtain for each mesoid the PET, and then the corresponding response time. Determining the worst case among these response times allow us to conclude on the schedulability of operation  $\tau_i$  w.r.t.  $\prec$ . When  $\tau_i$  is schedulable, we build the sequence  $\mathcal{L}_i^a$ , after  $\tau_i$  is scheduled, in order to check the schedulability of the next operation, and so on, otherwise the system is not schedulable.

#### 4.1 Scheduling algorithm

We assume that the first  $i-1$  operations with  $2 \leq i \leq n$  have already been scheduled, i.e. the sequence  $\mathcal{L}_{i-1}^a$  of operation  $\tau_{i-1}$  is known, and that we are about to schedule operation  $\tau_i$ , which is potentially schedulable, i.e. equation 1 holds.

Sequence  $\mathcal{L}_i^b = f(\mathcal{L}_{i-1}^a)$  of operation  $\tau_i$  is built thanks to index  $\psi$  on sequence  $\mathcal{L}_{i-1}^a$  of operation  $\tau_{i-1}$  without forgetting to start at the first available time unit rather than the first time unit as in [10]. Again this is due to the constraints on the system and the fact that no idle time is allowed: the start time of the first instance of operation  $\tau_i$  is at  $s_i^1 = s_{i-1}^1 + \Delta_{i-1}$ . The sequence  $\mathcal{L}_i^b$  consists of  $\sigma_i$   $T_i$ -mesoids  $\mathcal{M}_i^{b,k}$  with  $k = 1, \dots, \sigma_i$  since operation  $\tau_i$  may only be preempted by operations belonging to  $sp(\tau_i)$ . We can therefore determine the universes  $X_i^k \forall k \in \{1, \dots, \sigma_i\}$  when the sequence  $\mathcal{L}_{i-1}^a$  is known. The response time  $R_i^k$  of operation  $\tau_i$  in its  $k^{th}$  instance, i.e. in the  $k^{th}$   $T_i$ -mesoid will be obtained by summing  $C_i^k$  with all consumptions prior to  $C_i^k$  in the corresponding mesoid. The worst-case response time  $R_i$  of operation  $\tau_i$  will be given by  $R_i = \max_{\{1 \leq k \leq \sigma_i\}}(R_i^k)$ . This equation leads us to say that operation  $\tau_i$  is schedulable if and only if  $R_i \leq T_i$ . Again,  $\mathcal{L}_i^a = g(\mathcal{L}_i^b)$  will be deduced from sequence  $\mathcal{L}_i^b$  like  $\mathcal{L}_1^a = g(\mathcal{L}_1^b)$  in the previous section. For the sake of clarity, whenever there are two consecutive consumptions in the same *mesoid*, this amounts to considering only one consumption which is the sum of the previous consumptions.

Below, we present our scheduling algorithm which, for a given operation, on the one hand first determines the start time of its first instance relatively to  $\prec$ , then counts the exact number of preemptions in each of its instances, and on the other hand provides its PET in each of its instances in order to take into account the exact number of preemptions in the schedulability condition. It has the following twelve steps. Since the operation with the shortest period, namely operation  $\tau_1$ , is never preempted, the loop starts from the index of the operation with the second shortest period, namely operation  $\tau_2$ .

- 1: **for**  $i = 2$  to  $n$  **do**
- 2:   Compute the number  $\sigma_i$  of times that operation  $\tau_i = (C_i, T_i)$  has started in the hyper-period at level  $i$ .

- 3: Determine the start time of the first instance of operation  $\tau_i$ :  $s_i^1 = s_{i-1}^1 + \Delta_{i-1}$  where  $\Delta_{i-1}$  is the consumption before the first available time unit in the sequence  $\mathcal{L}_{i-1}^a$  of operation  $\tau_{i-1}$ .
- 4: Build the sequence  $\mathcal{L}_i^b = f(\mathcal{L}_{i-1}^a)$  of  $T_i$ -mesoids of operation  $\tau_i$  before it is scheduled. This construction consists of  $\sigma_i$   $T_i$ -mesoids  $\mathcal{M}_i^{b,k}$  with  $k = 1, \dots, \sigma_i$ , and is based on a modulo  $T_i$  arithmetic using index  $\psi$  on the sequence  $\mathcal{L}_{i-1}^a$  without forgetting to start at the first available time unit rather than the first time unit as in [10].
- 5: For each  $T_i$ -mesoid  $\mathcal{M}_i^{b,k}$  resulting from the previous step, build the corresponding universe  $X_i^k$  which consists of the ordered set of all availabilities of  $\mathcal{M}_i^{b,k}$ . Notice that this set corresponds to the set of all possible values that the PET  $C_i^k$  of operation  $\tau_i$  can take in  $\mathcal{M}_i^{b,k}$ .
- 6: Build all the equivalence classes for each universe  $X_i^k$ . An equivalence class of  $X_i^k$  is composed of the subset of availabilities determined by two consecutive consumptions in the associated mesoid  $\mathcal{M}_i^{b,k}$ .  $m \in \mathbb{N}$  in expression  $[m]^k$  denotes the subset of  $X_i^k$  composed of the availabilities which are preempted  $m$  times.
- 7: Compute both the exact number of preemptions and the PET  $C_i^k$  of operation  $\tau_i$  in each universe  $X_i^k$ ,  $1 \leq k \leq \sigma_i$ , resulting from the previous step thanks to the algorithm inlined in this step. Since  $\tau_i$  is potentially schedulable, i.e. its WCET  $C_i$  belongs to one and only one equivalence class  $[\theta_1]^k$  in each universe  $X_i^k$ , we must verify that it is actually schedulable given that some preemptions may occur. The recursive inflation of the execution time of the operation, due to preemptions, starts from the value of the WCET. Indeed, the current inflated WCET is obtained by adding the previous inflated WCET and the cost of preemptions incurred by this latter WCET. This explains the following fixed-point algorithm.

$$\begin{cases} \theta_0 = 0 \\ C_i^{k,0} = C_i \in [\theta_1]^k \\ C_i^{k,m} = C_i^{k,m-1} + (\theta_m - \theta_{m-1}) \cdot \alpha \in [\theta_m]^k \quad \forall m \geq 1 \end{cases}$$

This computation stops as soon as either the PET is reached, i.e. two consecutive values of  $C_i^{k,j}$ ,  $j \geq 1$  are equal, or there exists  $\mu_2 \geq 1$  such that  $C_i^{k,\mu_2} > \text{card}(X_i^k)$ . In

the latter case, operation  $\tau_i$  is not schedulable. In the first case,  $C_i^k = C_i + \sum_{j=1}^l (\theta_j - \theta_{j-1}) \cdot \alpha = C_i + N_p(\tau_i^k) \cdot \alpha$ .

- 8: Deduce the image  $\tau_i' = \pi(\tau_i) = ((C_i^1, C_i^2, \dots, C_i^{\sigma_i}), T_i)$  of operation  $\tau_i$  resulting from the previous step.
- 9: Determine the response time  $R_i^k$ ,  $1 \leq k \leq \sigma_i$  of operation  $\tau_i$  in its  $k^{\text{th}}$  instance, i.e. in the  $k^{\text{th}}$   $T_i$ -mesoid. This is obtained by summing  $C_i^k$  with all the consumptions prior to  $C_i^k$  in the corresponding mesoid. Deduce the worst-case response time  $R_i$  of operation  $\tau_i$ :  $R_i = \max_{\{1 \leq k \leq \sigma_i\}}(R_i^k)$ . Operation  $\tau_i$  is schedulable if and only if  $R_i \leq T_i$ .
- 10: If  $R_i \leq T_i$  then build the sequence  $\mathcal{L}_i^a = g(\mathcal{L}_i^b)$ , increment  $i$ , and go back to step 2 as long as there remain potentially schedulable operations in the system.
- 11: If  $R_i > T_i$ , then the system  $\{\tau_i = (C_i, T_i)\}_{1 \leq i \leq n}$  is not schedulable.
- 12: **end for**



Thanks to the above algorithm, a system  $\{\tau_i = (C_i, T_i)\}_{1 \leq i \leq n}$ , with precedence and strict periodicity constraints where no idle time is allowed and which takes into account the exact number of preemptions, is schedulable if and only if  $R_i \leq T_i$  for all  $i \in \{1, 2, \dots, n\}$ .

## 5 Conclusion and future work

We are interested in hard real-time systems with precedence and strict periodicity constraints where it is mandatory to satisfy these constraints. We are also interested in preemption which offers great advantages when seeking schedules. Since classical approaches are based on an approximation of the cost of the preemption in WCETs, possibly leading to an incorrect real-time execution, we proposed an approach that takes its exact cost into account. We proposed a scheduling algorithm which counts the exact number of preemptions for a given system and thus gives a stronger schedulability condition than those in the literature.

Currently, we are adding the latency constraints to our model and we are planning to study the same problem when jitter is allowed on the periods of operations and then, the complexity of our approach. Afterwards, because idle time may increase the possible schedules, we also plan to allow idle time, even though this would increase the complexity of the scheduling algorithm.

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